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MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

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Tuesday 12 November 2013 (morning)

Examination code

8	8	1	3	–	7	2	0	2

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

Consider the matrices $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$. Find the matrix $\boldsymbol{X}$ such that $\boldsymbol{AX} = \boldsymbol{B}$.

2. *[Maximum mark: 7]*

The fourth term in an arithmetic sequence is 34 and the tenth term is 76.

(a) Find the first term and the common difference. *[3]*

(b) The sum of the first n terms exceeds 5000. Find the least possible value of n. *[4]*

. . . .

3. *[Maximum mark: 7]*

Consider $f(x) = \ln x - e^{\cos x}$, $0 < x \le 10$.

(a) Sketch the graph of $y = f(x)$, stating the coordinates of any maximum and minimum points and points of intersection with the x-axis. *[5]*

(b) Solve the inequality $\ln x \le e^{\cos x}$, $0 < x \le 10$. *[2]*

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4. *[Maximum mark: 6]*

The duration of direct flights from London to Singapore in a particular year followed a normal distribution with mean μ and standard deviation σ.

92 % of flights took under 13 hours, while only 12 % of flights took under 12 hours 35 minutes.

Find μ and σ to the nearest minute.

5. *[Maximum mark: 5]*

At the start of each week, Eric and Marina pick a night at random on which they will watch a movie.

If they choose a Saturday night, the probability that they watch a French movie is $\frac{7}{9}$ and if they choose any other night the probability that they watch a French movie is $\frac{4}{9}$.

(a) Find the probability that they watch a French movie. *[3]*

(b) Given that last week they watched a French movie, find the probability that it was on a Saturday night. *[2]*

6. *[Maximum mark: 6]*

A complex number z is given by $z = \dfrac{a+\mathrm{i}}{a-\mathrm{i}}$, $a \in \mathbb{R}$.

(a) Determine the set of values of a such that

(i) z is real;

(ii) z is purely imaginary. *[4]*

(b) Show that $|z|$ is constant for all values of a. *[2]*

7. *[Maximum mark: 6]*

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a} = (3\cos\theta + 6)\boldsymbol{i} + 7\boldsymbol{j}$ and $\boldsymbol{b} = (\cos\theta - 2)\boldsymbol{i} + (1 + \sin\theta)\boldsymbol{j}$.

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular,

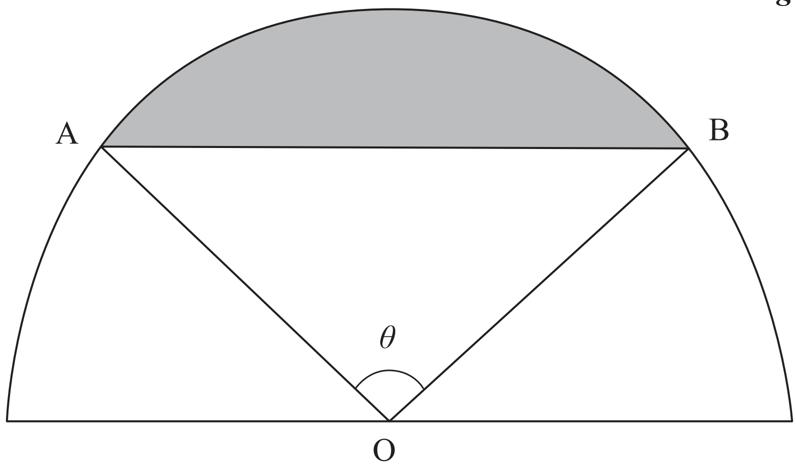
(a) show that $3\sin^2\theta - 7\sin\theta + 2 = 0$; *[3]*

(b) find the smallest possible positive value of θ. *[3]*

8. *[Maximum mark: 5]*

The diagram below shows a semi-circle of diameter 20 cm, centre O and two points A and B such that $A\hat{O}B = \theta$, where θ is in radians.

diagram not to scale



(a) Show that the shaded area can be expressed as $50\theta - 50\sin\theta$. *[2]*

(b) Find the value of θ for which the shaded area is equal to half that of the unshaded area, giving your answer correct to four significant figures. *[3]*

9. *[Maximum mark: 7]*

A line L_1 has equation $\boldsymbol{r} = \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

A line L_2 passing through the origin intersects L_1 and is perpendicular to L_1.

(a) Find a vector equation of L_2. *[5]*

(b) Determine the shortest distance from the origin to L_1. *[2]*

10. *[Maximum mark: 7]*

By using the substitution $x = 2\tan u$, show that $\int \frac{dx}{x^2\sqrt{x^2+4}} = \frac{-\sqrt{x^2+4}}{4x} + C$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 18]*

(a) The number of cats visiting Helena's garden each week follows a Poisson distribution with mean $\lambda = 0.6$.

Find the probability that

(i) in a particular week no cats will visit Helena's garden;

(ii) in a particular week at least three cats will visit Helena's garden;

(iii) over a four-week period no more than five cats in total will visit Helena's garden;

(iv) over a twelve-week period there will be exactly four weeks in which at least one cat will visit Helena's garden. *[9]*

(b) A continuous random variable X has probability distribution function f given by

$$f(x) = k \ln x \quad 1 \le x \le 3$$
$$f(x) = 0 \quad \text{otherwise}$$

(i) Find the value of k to six decimal places.

(ii) Find the value of $\mathrm{E}(X)$.

(iii) State the mode of X.

(iv) Find the median of X. *[9]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 20]*

(a) A particle P moves in a straight line with velocity $v\,\text{ms}^{-1}$. At time $t = 0$, P is at the point O and has velocity $12\,\text{ms}^{-1}$. Its acceleration at time t seconds is given by $\frac{\text{d}v}{\text{d}t} = 3\cos\frac{t}{4}\,\text{ms}^{-2}$, $(t \geq 0)$.

(i) Find an expression for the particle's velocity v, in terms of t.

(ii) Sketch a velocity/time graph for the particle for $0 \leq t \leq 8\pi$, showing clearly where the curve meets the axes and any maximum or minimum points.

(iii) Find the distance travelled by the particle before first coming to rest. *[8]*

(b) Another particle Q moves in a straight line with displacement s metres and velocity $v\,\text{ms}^{-1}$. Its acceleration is given by $a = -(v^2 + 4)\,\text{ms}^{-2}$, $(0 \leq t \leq 1)$. At time $t = 0$, Q is at the point O and has velocity $2\,\text{ms}^{-1}$.

(i) Show that the velocity v at time t is given by $v = 2\tan\left(\frac{\pi - 8t}{4}\right)$.

(ii) Show that $\frac{\text{d}v}{\text{d}s} = -\frac{(v^2 + 4)}{v}$.

(iii) Find the distance travelled by the particle before coming to rest. *[12]*

Turn over

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 22]*

A function f is defined by $f(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right),\ x \in \mathbb{R}$.

(a) (i) Explain why the inverse function f^{-1} does not exist.

(ii) Show that the equation of the normal to the curve at the point P where $x = \ln 3$ is given by $9x + 12y - 9\ln 3 - 20 = 0$.

(iii) Find the x-coordinates of the points Q and R on the curve such that the tangents at Q and R pass through $(0, 0)$. *[14]*

(b) The domain of f is now restricted to $x \geq 0$.

(i) Find an expression for $f^{-1}(x)$.

(ii) Find the volume generated when the region bounded by the curve $y = f(x)$ and the lines $x = 0$ and $y = 5$ is rotated through an angle of 2π radians about the y-axis. *[8]*

Please **do not** write on this page.

Answers written on this page
will not be marked.